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A Mathematical Integrated Model for Shop Scheduling Routing Considering Reliability and Maintenance under Uncertainty Conditions

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A B S T R A C T

This study presents an integrated mathematical model for shop routing and scheduling while considering machine reliability and maintenance under uncertainty conditions. The primary objective of the proposed model is to optimize production time and system costs through effective machine management, activity scheduling, and preventive maintenance planning. To achieve this objective, the mathematical model incorporates a set of decision variables and parameters, including machine-related costs, energy consumption costs, and maintenance and repair costs. Furthermore, the model includes constraints related to activity continuity, precise activity scheduling, compliance with time restrictions, costs associated with machine utilization, and machine maintenance levels. The proposed model is particularly applicable to manufacturing environments that require efficient management of resources and machinery and can assist production managers in reducing operational costs and production lead times. The model can be solved using numerical optimization algorithms, and its results can support optimal decision-making regarding scheduling, routing, and machine maintenance in manufacturing systems.

Keywords: *Mathematical model, scheduling, routing, reliability, maintenance and repair, uncertainty.*

1. Introduction

Modern manufacturing systems operate in an environment characterized by increasing product variety, shorter delivery lead times, resource limitations, machine degradation, and demand uncertainty. In such conditions, production managers are required to make several interdependent decisions simultaneously, including assigning operations to machines, sequencing jobs, determining production routes, coordinating transportation or movement between resources, and planning maintenance activities. Traditional production planning approaches often treat routing, scheduling, reliability, and maintenance as separate decision layers; however, this separation can lead to suboptimal decisions because machine availability, processing sequence, setup requirements, energy consumption, and preventive maintenance actions are strongly interconnected. Therefore, integrated mathematical models have become increasingly important for improving the operational performance of workshop and job-shop production systems, especially where machines are heterogeneous, resources are constrained, and uncertainty affects processing times, travel times, machine failures, and delivery commitments (Abedinnia et al., 2017; Fanjul-Peyro et al., 2017; Santos et al., 2017).

Scheduling has long been recognized as a central decision problem in production and operations management. In job-shop, flow-shop, flexible job-shop, and parallel-machine environments, the allocation of operations to machines and the sequencing of jobs directly influence makespan, tardiness, machine utilization, production cost, and service reliability. The complexity of these problems increases significantly when additional real-world constraints such as unrelated parallel machines, sequence-dependent setup times, batch processing, release times, buffer capacity, job rejection, dual human–machine resources, and multi-factory structures are incorporated into the model (Babaei & Behnamian, 2022; Fattahi et al., 2022; Havad & Sara, 2023; Mahimini & Bidgoli, 2023; Nikouei et al., 2024). These developments indicate that classical scheduling assumptions are no longer sufficient for representing contemporary manufacturing systems, where flexibility and operational realism must be embedded in the mathematical formulation.

In addition to scheduling decisions, routing decisions have become an essential component of integrated production systems. Routing is not limited to the external distribution of finished goods but may also involve the movement of jobs, materials, tools, operators, or transport resources inside or outside the production environment. The vehicle routing problem and its numerous variants, including routing with time windows, heterogeneous fleets, uncertain travel times, and setup-dependent production routing, provide a strong foundation for modeling such decisions (Baradaran et al., 2019; Vidal et al., 2020; Wu et al., 2022). In production systems where routing and scheduling interact, decisions about the production sequence influence routing feasibility, while routing constraints affect completion times and delivery performance. Consequently, integrated production scheduling and vehicle routing models have gained increasing attention because they provide a more realistic representation of manufacturing and logistics operations (Mohammadi et al., 2019; J. Wang et al., 2019; Zou et al., 2018).

The integration of scheduling and routing has been investigated in several production contexts, including open-shop environments, distributed production systems, multi-purpose machine environments, outbound vehicle routing, and production systems with time-window constraints. These studies show that integrated models can reduce total costs, improve delivery reliability, and increase coordination between production and logistics decisions (Berghman et al., 2023; Chevroton et al., 2021; Su et al., 2023). For example, integrated approaches allow production managers to simultaneously determine when each job should be processed, which machine should process it, and how the resulting outputs should be transferred or delivered. Such integration is particularly valuable when routing delays, transportation resources, or time-window penalties affect the overall objective function. Recent research has further emphasized energy-efficient integrated production scheduling and routing, reflecting the growing importance of sustainability and operational cost reduction in manufacturing systems (Lu et al., 2017; D. Wang et al., 2019; Yağmur & Kesen, 2024).

Despite these advances, many integrated scheduling-routing models still assume that machines are continuously available and fully reliable. This assumption is often unrealistic because industrial machines are subject to deterioration, random failures, performance degradation, and maintenance requirements. Reliability-oriented modeling is therefore essential for ensuring feasible and stable production schedules. Reliability affects not only the probability of completing operations without failure but also the timing and cost of maintenance interventions. If maintenance is ignored, an apparently optimal schedule may become infeasible or costly in practice because unexpected breakdowns can disrupt production flow, increase tardiness, and raise repair costs. Studies on reliability and maintenance policy systems have shown that maintenance planning must be connected to production decisions to enhance system availability and reduce operational risk (Bhatti et al., 2021; Gu et al., 2021; Guan et al., 2019).

Preventive maintenance is particularly important in complex workshop environments because it allows organizations to reduce the likelihood of machine failure before breakdowns occur. However, preventive maintenance also consumes time and resources and may temporarily remove machines from production. Therefore, an optimal model must balance the short-term cost of maintenance with the long-term benefits of improved reliability and reduced disruption. Integrated production scheduling and preventive maintenance planning models address this trade-off by jointly determining production sequences and maintenance actions (Ettahiry et al., 2022; Khaksari & Bidgoli, 2024; Kolus et al., 2020). When maintenance is considered together with job-shop routing and scheduling, the decision problem becomes more realistic but also more computationally challenging because machine condition, maintenance level, processing sequence, and resource allocation must be optimized simultaneously.

Uncertainty further increases the complexity of integrated routing, scheduling, and maintenance models. In real manufacturing environments, uncertainty may arise from machine breakdowns, processing-time variation, travel-time fluctuations, demand changes, order cancellations, random job arrivals, and resource availability. Robust and stochastic optimization approaches have been used to address uncertainty in routing and production systems, particularly where demand and travel times are not deterministic (Eshtehadi et al., 2017; Shahbazi & Vahidi, 2024; J. Wang et al., 2019). In dynamic job-shop environments, recent studies have also highlighted the role of data-driven, simulation-based, reinforcement learning, and transformer-guided methods for dealing with random arrivals, machine failures, and changing shop-floor conditions (Ji & Yan, 2023; Lassoued, 2026; Tariq et al., 2024; Xu et al., 2026). These approaches demonstrate that uncertainty must be explicitly incorporated into modern scheduling models rather than treated as an external disturbance.

The flexible job-shop scheduling problem is one of the most relevant frameworks for modeling contemporary production environments because each operation can often be processed by alternative machines. This flexibility creates opportunities for improved utilization and reduced completion time but also expands the decision space. When transportation resources, machine reliability, preventive maintenance, job rejection policies, and uncertainty are considered, the flexible job-shop scheduling problem becomes a multi-dimensional optimization problem (Berterottière et al., 2024; Soufi et al., 2023; Zhu et al., 2024). Distributed and multi-factory production settings further increase this complexity because jobs may be assigned across different factories or production units, and coordination between facilities becomes necessary (Bagheri-Rad & Behnamian, 2023; Rostami & Pour, 2022). These characteristics show the need for integrated models capable of capturing both production and resource interactions under realistic constraints.

Another important dimension in modern production scheduling is the presence of additional resources beyond machines. These may include workers, transporters, tools, shifts, support resources, or human–machine combinations. Research on personnel allocation, staff and machine shift scheduling, dual resource-constrained scheduling, and maintenance support resource scheduling confirms that production performance depends on the coordinated availability of multiple resource types (Babaei & Behnamian, 2022; Santos et al., 2017; Schulze & Zimmermann, 2017; Soufi et al., 2023). Although the present study focuses on machine allocation, reliability, routing, and maintenance, this broader literature underscores the importance of modeling resource constraints in an integrated way rather than optimizing each resource separately.

From a methodological perspective, integrated routing and scheduling problems are generally NP-hard and difficult to solve exactly for large-scale instances. Exact mathematical programming approaches, such as those implemented in GAMS, are useful for validating small-sized instances and obtaining benchmark solutions. However, their computational burden increases rapidly as the number of machines, jobs, constraints, and scenarios grows. Consequently, metaheuristic and hybrid algorithms have become common solution approaches for large and complex production optimization problems. Variable neighborhood search, cuckoo search, simulated annealing, memetic algorithms, multi-objective backtracking search, Lagrangian relaxation, and other evolutionary or hybrid methods have been successfully applied to scheduling, routing, and maintenance-related problems (Bagheri-Rad & Behnamian, 2023; Dahite et al., 2022; Lu et al., 2017; Wu et al., 2022; Zhu et al., 2024). These methods are especially useful when the model includes multiple objectives and nonlinear interactions among production, cost, reliability, and maintenance variables.

Multi-objective optimization is particularly appropriate for integrated workshop routing and scheduling because production managers rarely optimize a single performance criterion. In practice, minimizing makespan may conflict with minimizing energy consumption, reducing maintenance cost, preserving reliability, or avoiding time-window violations. Therefore, Pareto-based algorithms such as NSGA-II provide a suitable framework for generating a set of non-dominated solutions that reflect trade-offs among competing objectives. Previous studies have emphasized the need to consider multiple objectives in flexible job-shop scheduling, energy-efficient production, routing with time windows, and integrated maintenance planning (Dahite et al., 2022; Su et al., 2023; D. Wang et al., 2019; Yağmur & Kesen, 2024). In this context, a multi-objective metaheuristic can help decision-makers select solutions according to managerial priorities, such as lower cost, shorter production time, or higher reliability.

The literature also indicates a growing shift toward smart, dynamic, and data-supported scheduling systems. Industry 4.0 technologies, real-time manufacturing execution systems, big data analytics, and intelligent optimization methods are increasingly being used to improve scheduling responsiveness and shop-floor adaptability (Ji & Yan, 2023; Tariq et al., 2024). These developments are important because uncertainty and machine failures cannot always be fully predicted at the planning stage. Therefore, optimization models must be designed in a way that supports robustness, sensitivity analysis, and practical applicability under changing operating conditions. Simulation-based optimization and dynamic scheduling methods provide useful insights for handling uncertainty, but integrated mathematical models remain essential because they define the decision structure, objective functions, and constraints that guide algorithmic search (Lassoued, 2026; Shahbazi & Vahidi, 2024; Xu et al., 2026).

Although previous studies have made substantial contributions to production scheduling, routing, maintenance planning, and reliability modeling, several research gaps remain. Many studies focus on scheduling without explicitly incorporating routing decisions, while others address routing without sufficiently modeling machine reliability and preventive maintenance. Some studies consider maintenance and production jointly but do not integrate routing or uncertainty. Similarly, several models optimize cost or makespan but do not simultaneously include energy consumption, maintenance cost, fixed machine cost, and soft time-window penalties. This fragmentation limits the applicability of existing models to real workshop environments where routing, scheduling, reliability, and maintenance decisions are interdependent (Berghman et al., 2023; Berterottière et al., 2024; Khaksari & Bidgoli, 2024; Kolus et al., 2020). Therefore, there is a clear need for a comprehensive mathematical framework that integrates these elements within a unified multi-objective decision model.

Accordingly, the present study contributes to the literature by developing an integrated mathematical model for workshop routing and scheduling that simultaneously considers machine reliability, preventive maintenance, energy consumption, production timing, machine-related costs, and uncertainty. The model is designed to reflect practical production constraints, including machine assignment, operation sequencing, setup requirements, time-window restrictions, maintenance levels, and cost components. It also responds to recent calls for more realistic, scalable, and computationally efficient optimization approaches in complex manufacturing systems (Nikouei et al., 2024; Yağmur & Kesen, 2024; Zhu et al., 2024). By solving small-sized instances through GAMS and large-sized instances through NSGA-II, the study provides both exact validation and scalable metaheuristic performance evaluation. This combined approach strengthens the practical relevance of the proposed model and supports its applicability in production environments where exact methods become computationally prohibitive.

The aim of this study is to develop and evaluate an integrated mathematical model for workshop routing and scheduling under uncertainty by simultaneously considering machine reliability and preventive maintenance in order to minimize total production time and total system cost.

2. Methods and Materials

This study is applied in terms of its objective and descriptive–analytical in nature. It develops an integrated mathematical model for shop routing and scheduling through the use of library-based research and computational methods. The data analysis process is conducted at two levels. First, the mathematical model is solved using the GAMS software package to validate the model and obtain exact solutions. Subsequently, to address large-scale and real-world problem instances, the multi-objective metaheuristic algorithm NSGA-II (Non-dominated Sorting Genetic Algorithm II) is implemented in the MATLAB environment. This approach enables the extraction of the Pareto frontier and the performance of sensitivity analyses on key parameters to evaluate the robustness and efficiency of the proposed model under uncertainty conditions.

The central problem addressed in this research is the optimal allocation of machines to production activities and the scheduling of these activities while simultaneously considering machine reliability and preventive maintenance under uncertain scenarios. The model seeks to optimize two primary objective functions: minimizing total production completion time and reducing total system costs, including energy consumption costs, maintenance and repair costs, fixed operating costs, and penalty costs associated with violations of soft time-window constraints.

The principal assumptions and constraints of the proposed model include a fixed number of machines, compliance with specified time windows, the requirement to maintain a predetermined reliability level for equipment, and the consideration of sequence- and machine-dependent setup times. These assumptions and constraints are incorporated into a comprehensive and coherent mathematical programming framework.

3. Findings and Results

To evaluate the performance of the proposed NSGA-II algorithm in small-scale problem instances, five numerical examples were randomly generated and solved using both the NSGA-II algorithm and the GAMS optimization software. The purpose of this comparison was to assess the accuracy and efficiency of the proposed algorithm by comparing its objective function values with the exact solutions obtained from GAMS.

The comparison results demonstrated that the objective function values generated by NSGA-II were highly consistent with those obtained from GAMS. The maximum deviation observed among all test problems was only 2.92%, indicating that the proposed algorithm is capable of producing near-optimal solutions with a high degree of accuracy. These findings confirm the suitability of NSGA-II for solving small-scale instances of the integrated routing and scheduling problem.

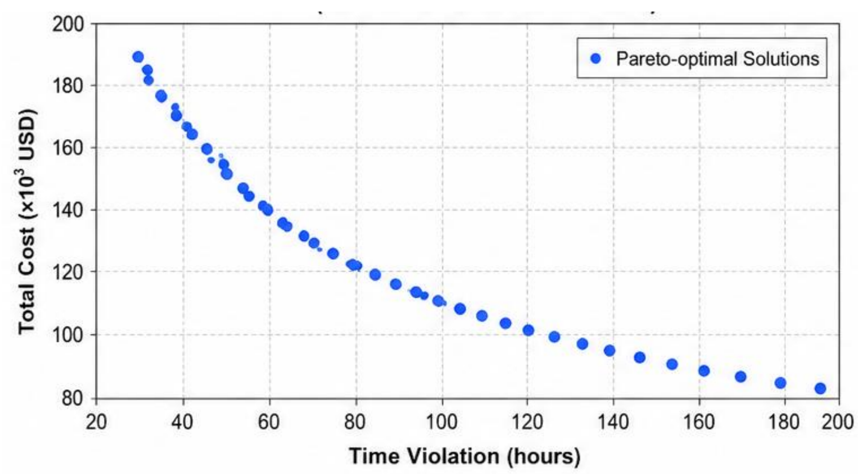
Table 1

Performance Evaluation of NSGA-II for Small-Sized Problems

Example No.	GAMS Time-Deviation Objective	NSGA-II Time-Deviation Objective	Deviation (%)	GAMS Cost Objective	NSGA-II Cost Objective	Deviation (%)
1	0	0	0.00	32.2	32.3	0.31
2	0	0	0.00	46.3	46.9	1.30
3	0	0	0.00	76.1	77.6	1.97
4	3	3	0.00	99.8	102.1	2.30
5	4	4	0.00	123.4	127.0	2.92

Figure 1

Comparison of Cost Objective Function Values Obtained by GAMS and NSGA-II for Small-Sized Problems

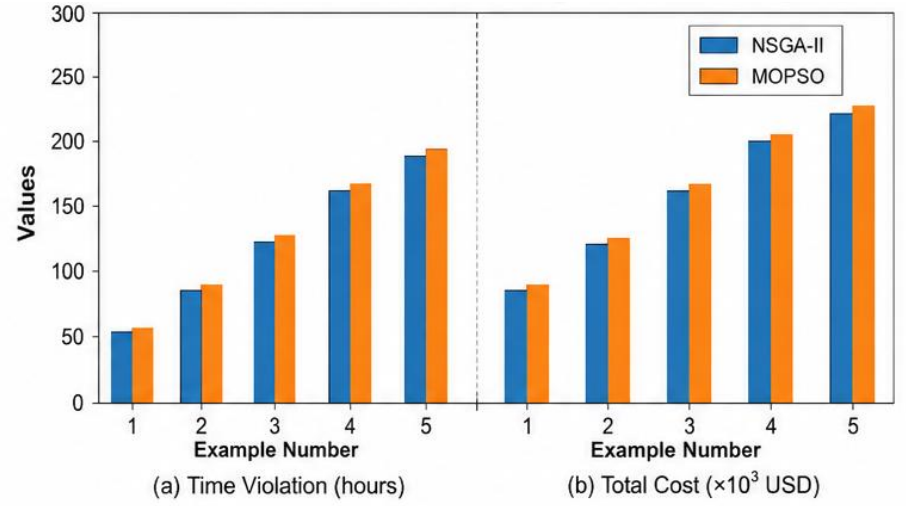


The results reported in Table 1 reveal that the differences between the solutions obtained by NSGA-II and GAMS are negligible. Furthermore, the time-deviation objective function was reproduced exactly in all five examples. Therefore, the proposed metaheuristic algorithm can reliably approximate exact solutions while maintaining a significantly lower computational burden.

A further comparison was conducted regarding computational time. The results showed that the NSGA-II algorithm required substantially less computational time than GAMS. This difference becomes particularly important when dealing with larger optimization problems where exact solution approaches become computationally expensive. The reduction in computational effort can be attributed to the evolutionary search mechanism of NSGA-II, which efficiently explores the solution space without exhaustive enumeration.

Figure 2

Comparison of Solution Times Obtained by NSGA-II and GAMS



To assess the scalability of the proposed approach, several large-scale numerical instances were generated using randomly produced datasets. Each problem instance was solved thirty times using the NSGA-II algorithm to ensure statistical reliability and result stability. The best solution obtained from NSGA-II was subsequently compared with the solution generated by the MOPSO algorithm.

The comparison focused on both objective functions, namely total cost and total time-window violation. The results indicate that NSGA-II performs competitively in large-scale environments and is capable of generating high-quality solutions. The maximum deviation between the solutions obtained by NSGA-II and MOPSO was limited to 2.95% for the time-violation objective function and 2.48% for the cost objective function, demonstrating the robustness and effectiveness of the proposed algorithm.

Table 2

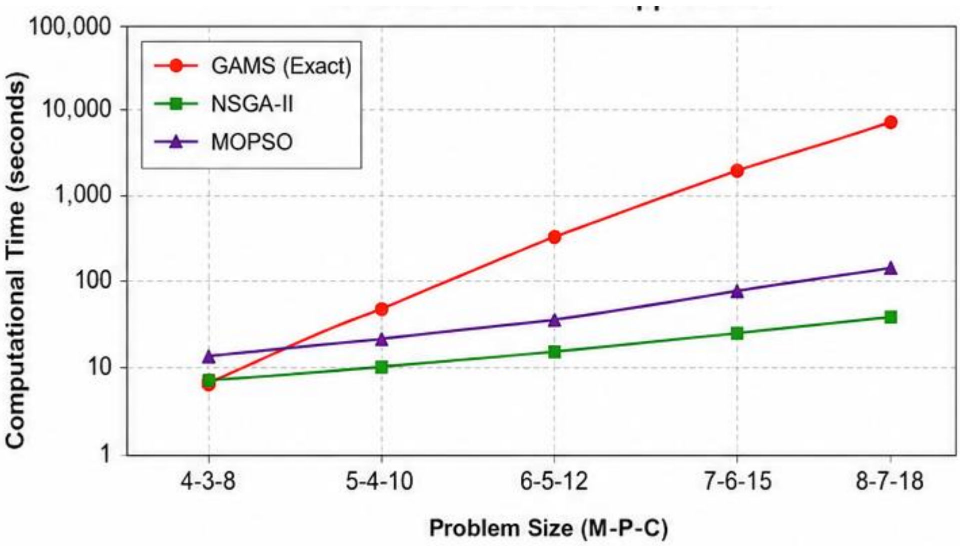
Performance Evaluation of NSGA-II Compared with MOPSO for Large-Sized Problems

Example No.	M	P	C	NSGA-II Time Violation	MOPSO Time Violation	Deviation (%)	NSGA-II Cost	MOPSO Cost	Deviation (%)
1	2	5	20	52.3	52.7	0.76	88.5	88.9	0.45
2	2	6	25	86.7	87.7	1.15	122.9	123.9	0.81
3	3	7	30	125.2	127.2	2.08	161.4	164.0	1.61
4	3	8	35	163.5	167.5	2.26	199.7	203.4	1.85
5	4	10	40	189.7	195.7	2.95	225.9	231.5	2.48

The findings confirm that NSGA-II maintains its effectiveness even as problem dimensions increase substantially. The relatively small deviations observed between NSGA-II and MOPSO indicate that both algorithms are capable of identifying high-quality solutions; however, NSGA-II achieves these results with a superior balance between solution quality and computational efficiency. Consequently, the proposed algorithm can be considered a suitable optimization tool for large-scale integrated routing and scheduling problems involving reliability and maintenance considerations.

Figure 3

Comparison of NSGA-II and MOPSO Performance in the Cost Objective Function



To further evaluate algorithmic efficiency, the computational times of NSGA-II, MOPSO, and GAMS were compared using problem instances of increasing size. The comparison was conducted to identify the most efficient solution approach for practical implementation.

Table 3

Comparison of Computational Times for Different Solution Methods

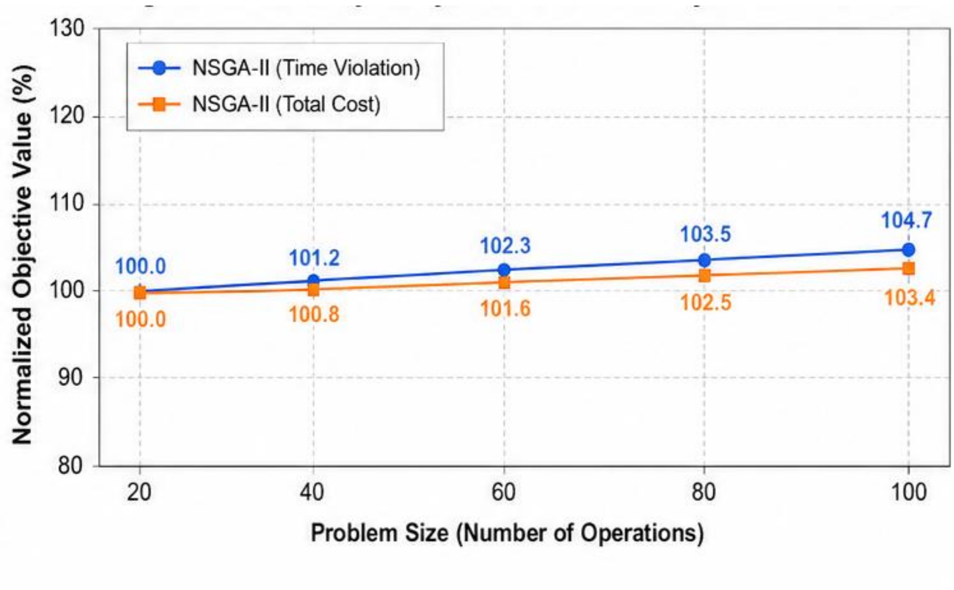
M	P	C	GAMS (s)	NSGA-II (s)	MOPSO (s)
4	3	8	8	10	15
5	4	10	42	14	25
6	5	12	321	22	45
7	6	15	1865	39	78
8	7	18	7201	58	142

The results presented in Table 3 clearly demonstrate the computational advantages of the NSGA-II algorithm. As the size of the problem increases, the solution time required by GAMS grows exponentially, reaching 7,201 seconds for the largest test instance. In contrast, NSGA-II required only 58 seconds for the same problem, while MOPSO required 142 seconds. These findings indicate that NSGA-II significantly outperforms both alternative approaches in terms of computational efficiency.

The superior performance of NSGA-II can be attributed to several factors. First, its evolutionary search mechanism employs genetic operators and non-dominated sorting strategies that efficiently explore the solution space without requiring exhaustive enumeration. Second, unlike GAMS, which relies on exact mathematical optimization procedures, NSGA-II utilizes approximate search techniques that substantially reduce computational complexity. Third, the algorithm effectively manages a population of solutions and simultaneously explores multiple regions of the search space, accelerating convergence toward high-quality Pareto-optimal solutions. Finally, NSGA-II maintains diversity among candidate solutions while achieving rapid convergence, enabling it to identify competitive solutions in significantly shorter execution times than both MOPSO and GAMS.

Figure 4

Comparison of Computational Times among NSGA-II, MOPSO, and GAMS



Overall, the computational experiments demonstrate that NSGA-II provides an effective balance between solution quality and computational efficiency. The algorithm achieves near-optimal objective values while requiring substantially less computational time, making it a practical and reliable approach for solving large-scale integrated routing and scheduling problems under uncertainty with reliability and maintenance considerations.

4. Discussion and Conclusion

The primary objective of this study was to develop and evaluate an integrated mathematical model for workshop routing and scheduling under uncertainty while simultaneously considering machine reliability and preventive maintenance activities. The proposed model aimed to minimize total production time and total system cost by integrating machine allocation, activity scheduling, routing decisions, reliability requirements, energy consumption, and maintenance planning within a unified optimization framework. The computational results obtained from both small-sized and large-sized problem instances demonstrate that the proposed model and solution approach are effective in generating high-quality solutions while maintaining computational efficiency. More specifically, the results revealed that the NSGA-II algorithm was capable of producing near-optimal solutions with very low deviations from exact solutions generated by GAMS in small-scale problems and showed competitive performance compared with alternative metaheuristic approaches in large-scale problem instances.

One of the most important findings of the study was the high accuracy of the NSGA-II algorithm in solving small-sized instances. The results showed that the maximum deviation between NSGA-II and the exact solutions obtained through GAMS did not exceed 2.92%, while deviations in the time-violation objective function were negligible. These findings indicate that the proposed metaheuristic algorithm is capable of accurately approximating optimal solutions. This result is consistent with previous studies that have emphasized the effectiveness of evolutionary and population-based algorithms in solving complex scheduling and routing problems where exact approaches become computationally expensive (Lu et al., 2017; Su et al., 2023; Wu et al., 2022). The small deviations observed between the proposed approach and exact optimization methods suggest that the algorithm successfully balances exploration and exploitation within the search space, enabling it to identify high-quality solutions without requiring exhaustive enumeration.

The results further demonstrated that the proposed integrated model effectively captures the interactions among production scheduling, routing decisions, reliability requirements, and maintenance activities. This finding is particularly important because many existing studies have treated these decision areas separately, potentially overlooking important operational interdependencies. The superior performance of the proposed framework supports the argument that integrated optimization approaches provide more realistic representations of manufacturing systems than isolated scheduling or routing models. Similar conclusions have been reported in studies investigating integrated production scheduling and vehicle routing problems, which showed that coordinated decision-making improves operational efficiency and reduces overall system costs (Berghman et al., 2023; Mohammadi et al., 2019; Zou et al., 2018). Likewise, research on integrated scheduling and routing has demonstrated that synchronization between production and transportation decisions can significantly improve resource utilization and delivery performance (Chevroton et al., 2021; Su et al., 2023; Yağmur & Kesen, 2024).

Another significant finding concerns the incorporation of reliability and preventive maintenance into the scheduling and routing framework. The results indicate that considering maintenance decisions simultaneously with production planning contributes to improved system performance and solution feasibility. By explicitly incorporating machine reliability and preventive maintenance requirements, the proposed model avoids the unrealistic assumption of continuous machine availability. This finding supports earlier studies that emphasized the importance of integrating maintenance planning with production scheduling in order to improve system reliability and reduce disruptions caused by equipment failures (Ettahiry et al., 2022; Gu et al., 2021; Kolus et al., 2020). Furthermore, the results align with the findings of Bhatti et al., who demonstrated that maintenance policies significantly influence system reliability and long-term operational performance (Bhatti et al., 2021). The present study extends these findings by showing that maintenance considerations remain important even when routing decisions and uncertainty factors are simultaneously incorporated into the optimization process.

The effectiveness of reliability-oriented scheduling observed in this study may be explained by the ability of preventive maintenance activities to reduce unexpected machine breakdowns and stabilize production flows. When maintenance interventions are optimally scheduled, machines remain available during critical production periods, thereby reducing delays and improving schedule robustness. Similar observations were reported in studies examining maintenance scheduling and reliability optimization, which found that maintenance planning contributes to higher equipment availability and lower production interruptions (Dahite et al., 2022; Guan et al., 2019; Khaksari & Bidgoli, 2024). Therefore, the present findings reinforce the notion that maintenance should not be considered merely as a support activity but rather as a strategic decision variable within integrated production optimization models.

The study also confirmed the effectiveness of the proposed model under uncertainty conditions. Manufacturing systems are inherently exposed to uncertainty arising from machine failures, variable processing times, fluctuating demand, transportation disruptions, and operational disturbances. The results indicate that the proposed framework remains capable of generating high-quality solutions despite these uncertainties. This finding is consistent with previous research emphasizing the necessity of incorporating uncertainty into production scheduling and routing models to improve their practical applicability (Baradaran et al., 2019; Eshtehadi et al., 2017; Shahbazi & Vahidi, 2024). By explicitly considering uncertainty, the proposed model provides more realistic schedules and routing plans that are less vulnerable to disruptions and unexpected operational events.

The strong performance of the proposed approach under uncertain conditions may also be interpreted in light of recent developments in dynamic scheduling research. Studies focusing on big data-driven scheduling, Industry 4.0 technologies, and intelligent manufacturing systems have highlighted the need for adaptive optimization frameworks capable of responding to changing shop-floor conditions (Ji & Yan, 2023; Tariq et al., 2024). Similarly, recent research on reinforcement learning and transformer-guided scheduling methods has emphasized the importance of handling machine failures and dynamic job arrivals in modern production environments (Lassoued, 2026; Xu et al., 2026). Although the present study adopts a mathematical programming and metaheuristic optimization perspective rather than a machine-learning-based approach, the results support the broader conclusion that uncertainty-aware optimization methods provide superior decision support compared with deterministic planning frameworks.

A further noteworthy finding concerns the scalability of the proposed solution approach. The comparison between

NSGA-II and MOPSO in large-scale problem instances demonstrated that the deviations between the two methods remained relatively small, with the largest difference remaining below 3%. This result indicates that NSGA-II is capable of generating solutions of comparable quality while maintaining strong computational performance. Similar findings have been reported in studies employing multi-objective metaheuristic algorithms for flexible job-shop scheduling and integrated routing problems, where Pareto-based evolutionary methods achieved high-quality solutions across multiple objective functions (Lu et al., 2017; Yağmur & Kesen, 2024; Zhu et al., 2024). The ability of NSGA-II to maintain solution quality as problem size increases highlights its suitability for practical industrial applications.

The computational-time analysis provides additional evidence supporting the effectiveness of the proposed methodology. The results revealed that NSGA-II required substantially less computational time than GAMS and consistently outperformed MOPSO as problem size increased. For the largest test instances, the computational advantage of NSGA-II became particularly significant. This finding is consistent with the broader optimization literature, which suggests that exact mathematical programming methods often encounter exponential growth in computational effort when solving large-scale combinatorial optimization problems (Abedinnia et al., 2017; Fanjul-Peyro et al., 2017). In contrast, population-based evolutionary algorithms can efficiently explore large solution spaces without evaluating every possible combination of decision variables. The observed computational efficiency of NSGA-II is therefore aligned with previous studies demonstrating the advantages of metaheuristic algorithms in large-scale production optimization contexts (Bagheri-Rad & Behnamian, 2023; Wu et al., 2022; Zhu et al., 2024).

The superior computational performance of NSGA-II can also be attributed to its non-dominated sorting mechanism and diversity-preservation strategy. These characteristics enable the algorithm to simultaneously search multiple regions of the solution space while avoiding premature convergence. Similar advantages have been discussed in studies addressing multi-objective scheduling, routing, and energy-efficient production optimization (Lu et al., 2017; D. Wang et al., 2019; Yağmur & Kesen, 2024). Consequently, the results of this study further validate the suitability of NSGA-II as a practical solution method for integrated workshop routing and scheduling problems involving multiple conflicting objectives.

Another important contribution of the present study is the inclusion of energy-related and operational cost components within the objective function. Modern manufacturing organizations face increasing pressure to reduce energy consumption while maintaining productivity and service quality. By integrating energy consumption, maintenance costs, machine utilization costs, and time-window penalties into a unified objective framework, the proposed model supports more comprehensive decision-making. This finding is consistent with recent studies emphasizing the importance of sustainable and energy-efficient scheduling approaches in manufacturing systems (Lu et al., 2017; Rostami & Pour, 2022; Yağmur & Kesen, 2024). The results suggest that organizations can achieve improvements in both economic and operational performance when these cost dimensions are jointly optimized.

Furthermore, the integrated nature of the proposed model aligns with contemporary trends in smart manufacturing and Industry 4.0. Recent studies have emphasized the need to simultaneously coordinate multiple operational decisions, including production planning, routing, maintenance, workforce allocation, and resource management (Berterottière et al., 2024; Santos et al., 2017; Schulze & Zimmermann, 2017). The present findings support this perspective by demonstrating that integrated optimization can provide substantial benefits over isolated decision-making approaches. In particular, the coordination of routing, scheduling, and maintenance decisions contributes to improved resource utilization, reduced operational costs, and enhanced system reliability.

Overall, the findings indicate that the proposed integrated mathematical model successfully addresses several important challenges in modern production systems. The model effectively incorporates routing decisions, machine scheduling, reliability requirements, preventive maintenance planning, and uncertainty within a unified optimization framework. The computational results confirm both the effectiveness of the mathematical formulation and the efficiency of the NSGA-II solution approach. Consequently, the study contributes to the growing body of literature on integrated production optimization and provides a practical framework for supporting decision-making in complex manufacturing environments.

One limitation of the present study is that the computational experiments were conducted using randomly generated problem instances rather than real industrial datasets. Although randomly generated benchmarks are commonly used to evaluate optimization algorithms, the use of actual production data could provide a more comprehensive assessment of the model's practical applicability. Additionally, uncertainty was represented through predefined parameters and assumptions, whereas real manufacturing environments may experience more complex and dynamic uncertainty patterns. Another limitation is that the study focused primarily on machine-related reliability and maintenance considerations and did not explicitly incorporate workforce availability, operator learning effects, supplier disruptions, or inventory-related uncertainties.

Future studies may extend the proposed model by incorporating real-time decision-making mechanisms and dynamic rescheduling capabilities. The integration of machine learning, reinforcement learning, digital twins, and Industry 4.0 technologies could further enhance the adaptability of the optimization framework. Researchers may also investigate multi-echelon supply chain environments in which production scheduling, maintenance planning, inventory management, and distribution routing are optimized simultaneously. Furthermore, future work could consider stochastic maintenance durations, carbon-emission objectives, workforce constraints, and multi-factory production networks to improve model realism and broaden its applicability across different industrial sectors.

Manufacturing managers can utilize the proposed framework as a decision-support tool for coordinating production scheduling, routing activities, and preventive maintenance planning. The results suggest that organizations can reduce production costs, improve resource utilization, and increase operational reliability by adopting integrated optimization approaches instead of managing these decisions independently. Industrial practitioners should prioritize preventive maintenance planning within scheduling processes and employ advanced metaheuristic optimization techniques when dealing with large-scale production systems. The implementation of such integrated models can support more efficient operations, enhance responsiveness to uncertainty, and improve the overall competitiveness of manufacturing organizations.

Authors' Contributions

Authors contributed equally to this article.

Declaration

In order to correct and improve the academic writing of our paper, we have used the language model ChatGPT.

Transparency Statement

Data are available for research purposes upon reasonable request to the corresponding author.

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Declaration of Interest

The authors report no conflict of interest.

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Ethics Considerations

In this research, ethical standards including obtaining informed consent, ensuring privacy and confidentiality were considered.

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