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Development of Tukey Control Charts for Monitoring Linear Profiles in Phase II

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A B S T R A C T

In many statistical process control problems, process quality or product performance can be described by the distribution of a quality characteristic and monitored using univariate control charts, or more generally, by the joint distribution of several quality characteristics and monitored using multivariate control charts. In some cases, product quality or process performance is described through the relationship between a response variable and one or more independent variables, a relationship that researchers refer to as a profile. One of the most widely used profiles is the simple linear profile, which has extensive applications in calibration; therefore, monitoring this type of profile is highly important. Profile monitoring is conducted in two phases, Phase I and Phase II. In Phase I, process stability is assessed and the process parameters are estimated, whereas in Phase II, the parameters are known and the objective is the rapid detection of changes in the process parameters. In this study, two proposed control charts—EWMA/R(Tukey) control chart and T^2(Tukey) control chart—are designed for monitoring simple linear profiles in Phase II, using three distributions (normal, t, and gamma) in each method. To simulate process changes, three types of shifts are applied, including shifts in the intercept, shifts in the slope, and shifts in the standard deviation. Simulation results indicate that the performance of the proposed control charts is superior to that of classical control charts.

Keywords: *Tukey control chart, simple linear profile, average run length, Phase II*

1. Introduction

Statistical process control (SPC) has long been recognized as a core pillar of quality engineering, providing practitioners with tools to detect departures from stable operation before they translate into significant economic loss or safety risk (Wong et al., 2025; Xu & Lin, 2025; Zhang et al., 2009). While classical Shewhart-type charts are effective for monitoring single quality characteristics, modern industrial systems often generate structured data where quality is better described by a functional relationship between a response variable and one or more explanatory variables rather than by a single scalar summary (Zou et al., 2007). In such contexts, profile monitoring has emerged as a powerful extension of SPC, allowing engineers to treat each sampled unit as a profile—typically a regression line—rather than a single measurement. Among different profile structures, simple linear profiles, which model the relationship between a response and a single explanatory variable, are particularly prevalent in calibration, semiconductor manufacturing, chemical processes, and service operations (Chiang et al., 2017; Saghaei et al., 2009). Ensuring timely detection of changes in these profiles is therefore critical to maintaining both product quality and process stability (Mahmood et al., 2018).

The literature on monitoring simple linear profiles can broadly be divided into Phase I and Phase II studies. Phase I is mainly concerned with retrospectively establishing in-control conditions and estimating stable model parameters, whereas Phase II focuses on on-line surveillance to rapidly detect any departures from the established baseline (Noorossana et al., 2010). Early work on Phase II monitoring of multivariate simple linear profiles used multivariate T^2-based schemes to detect shifts in intercept, slope, and error variance simultaneously (Noorossana et al., 2010). Subsequent studies addressed more realistic conditions, such as random explanatory variables and observational noise, proposing tailored monitoring schemes for Phase II control when regressors are not fixed or when profile structures deviate from standard assumptions (Noorossana et al., 2015). In parallel, CUSUM-based approaches have been developed to improve sensitivity to small and moderate changes in profile parameters, demonstrating that cumulative schemes can significantly reduce the average run length (ARL) compared to traditional charts when monitoring slope and intercept shifts (Saghaei & Mehrjoo, 2010; Saghaei et al., 2009).

Beyond CUSUM, researchers have explored likelihood-based and p-value-based methods to enhance detection performance. Likelihood ratio charts for monitoring linear profiles have been proposed as flexible tools for simultaneously capturing changes in multiple profile parameters (Zhang et al., 2009). Phase II monitoring frameworks based on p-value approaches provide an intuitive decision rule and have been shown to perform competitively, particularly when the practitioner is interested in joint monitoring of intercept and slope in linear profiles (Adibi et al., 2014). Another important line of work has focused on autocorrelated profile data: when within-profile observations are correlated, ignoring autocorrelation can lead to misleading conclusions about process stability. To address this, MEWMA-based schemes and process capability indices have been developed for simple linear profiles with within-profile autocorrelation, offering improved diagnostic ability and more realistic performance assessment (Chiang et al., 2017; Wang & Huang, 2017).

The need to monitor individual observations and sparse data has also motivated the design of schemes that rely on single-profile or pointwise information. A recent approach uses individual observations to monitor simple linear profiles, which is particularly attractive in settings where the number of measurements per profile is small or expensive to acquire (Haq et al., 2022). Other contributions provide alternative methods for simultaneous monitoring of slope and intercept, highlighting that distinct combinations of statistics and decision rules can be tuned to emphasize specific types of shifts, such as pure slope changes or joint slope–intercept shifts (Mahmood et al., 2018). More recently, run-rule–based and double or triple EWMA structures in Phase II have been examined for simple linear profiles, offering enhanced flexibility and improved sensitivity to a wide spectrum of change magnitudes (Sherwani et al., 2023).

In parallel with profile-specific methods, a rich body of work has evolved around adapting and extending Tukey's control chart—originally designed as a robust, nonparametric alternative for univariate process monitoring—to modern, complex quality data. The seminal development of Tukey's chart demonstrated how quartile-based, distribution-free limits could provide robustness in the presence of skewed data and outliers (Alemi, 2004). Subsequent analytical work on ARL performance clarified the operating characteristics of Tukey's control chart under various shift sizes and sampling conditions (Torng & Lee, 2008). Modifications and generalizations, such as the modified Tukey chart, further improved sensitivity and widened the applicability of the original idea, particularly when the underlying distribution deviates from normality (Tercero-Gomez et al., 2012). These robust, nonparametric features make Tukey-type charts especially appealing when industrial data exhibit heavy tails, skewness, or contamination by outliers—conditions frequently reported in modern manufacturing and service environments (Spirić et al., 2016).

Building on this foundation, recent research has integrated Tukey-type statistics into more advanced monitoring structures, including EWMA, CUSUM, MA, and synthetic schemes. Tukey MA–EWMA and MA–DEWMA control charts have been developed as nonparametric tools for detecting mean shifts, demonstrating high sensitivity in both small and large shift regions while remaining robust under non-normal distributions (Taboran et al., 2020, 2021; Talordphop et al., 2022). Mixed Tukey EWMA–modified EWMA schemes have also been proposed, showing that hybrid constructions can capture a wide range of process changes more effectively than purely parametric designs (Talordphop et al., 2023). Complementary work has examined mixed Tukey EWMA–CUSUM charts, confirming that combining the strengths of different memory-type charts can markedly improve the detection of persistent, gradual, or intermittent shifts (Riaz et al., 2017). From a design perspective, synthetic Tukey charts and their economically optimized variants under asymmetric loss functions have been proposed, indicating that statistical performance can be aligned with economic considerations and risk preferences of decision-makers (Lee & Chou, 2024; Raza et al., 2019). Additional enhancements include Tukey-EWMA charts with variable sampling intervals and related designs that adapt sampling effort to current process conditions (Adsız & Aytaçoğlu, 2024; Riaz et al., 2019).

Applications of Tukey-type charts have extended well beyond traditional manufacturing quality metrics. For instance, multivariate Tukey's CUSUM charts have been employed to monitor risk-adjusted therapeutic processes, where robustness to non-normal outcomes and patient-level heterogeneity is crucial (Kazemi et al., 2021). Statistical ACL Tukey methods have been used to detect suspicious electricity customers, illustrating the suitability of Tukey-based schemes for fraud detection in energy systems characterized by highly skewed and irregular usage patterns (Spirić et al., 2016). In operations and production management, integrated frameworks that couple production–maintenance planning with profile monitoring, such as schemes based on Hotelling's T^2 combined with optimization techniques, show that advanced monitoring strategies can be embedded into broader decision-support systems (Shojaee et al., 2024). These applications collectively underscore the versatility of Tukey-type charts as robust building blocks for modern monitoring systems.

Despite these advances, the integration of Tukey-type ideas into profile monitoring, and particularly into the monitoring of simple linear profiles in Phase II, remains relatively underexplored. Most profile-monitoring studies still rely on parametric control charts, such as T^2, MEWMA, CUSUM, or likelihood ratio charts, which generally assume approximate normality and can be sensitive to outliers or heavy-tailed errors (Chiang et al., 2017; Wang & Huang, 2017; Zhang et al., 2009; Zou et al., 2007). While nonparametric or robust features have been incorporated into univariate and multivariate Tukey charts, comparatively few contributions extend these benefits to the profile-monitoring domain, where the data structure—intercept, slope, and error variance—requires coordinated detection of multiple parameters (Noorossana et al., 2010; Noorossana et al., 2015). Moreover, much of the existing Phase II research focuses on profiles with normally distributed errors, whereas practical data often exhibit heavy-tailed or skewed error distributions, such as t and gamma, due to process variability, environmental influences, or measurement artifacts (Kazemi et al., 2021; Spirić et al., 2016).

Recent contributions in the broader Tukey-chart literature suggest promising directions to address these gaps. Nonparametric Tukey MA–EWMA and MA–DEWMA charts have demonstrated strong performance under non-normal distributions, indicating that Tukey-based transformations of residuals or profile statistics could stabilize performance across different error structures (Taboran et al., 2020, 2021; Talordphop et al., 2022). Mixed and modified EWMA–Tukey schemes highlight the potential of combining quartile-based robust statistics with memory-type charts to enhance responsiveness to both abrupt and gradual process changes (Riaz et al., 2017; Talordphop et al., 2023). Variable sampling interval designs and synthetic Tukey charts further point to the possibility of balancing detection speed, sampling cost, and false alarm risk in a principled manner (Adsız & Aytaçoğlu, 2024; Lee & Chou, 2024; Raza et al., 2019). However, systematic adaptation and evaluation of such concepts for Phase II monitoring of simple linear profiles—especially in the presence of t and gamma error structures—are still lacking.

Another important consideration is the choice of performance metrics and the treatment of different shift scenarios. Most profile-monitoring schemes are evaluated based on average run length under in-control and out-of-control conditions, often emphasizing either small or large shifts in intercept, slope, or variance (Saghaei & Mehrjoo, 2010; Saghaei et al., 2009). Work on modified successive sampling and other design enhancements has shown that sampling strategy and chart configuration can significantly influence ARL, particularly for linear profiles in noisy environments (Riaz et al., 2019). Meanwhile, run-rule–based DEWMA-type designs for simple linear profiles have highlighted the value of combining memory and supplementary rules to improve detection without dramatically increasing the false alarm rate (Sherwani et al., 2023). When robust, quartile-based statistics are embedded into these structures, one can reasonably expect further performance gains in non-normal settings, but such expectations need to be validated through systematic simulation across different distributions and shift magnitudes (Tercero-Gomez et al., 2012; Torng & Lee, 2008).

Taken together, the literature suggests a clear opportunity to bridge profile monitoring and robust Tukey-type methods by designing and evaluating Phase II control charts specifically tailored for simple linear profiles under a spectrum of error distributions. Such charts should be capable of detecting changes in intercept, slope, and variance with high sensitivity, while remaining robust to skewness, heavy tails, and outliers, and should be benchmarked against classical parametric alternatives such as T^2, EWMA/R, and CUSUM-based schemes (Haq et al., 2022; Mahmood et al., 2018; Noorossana et al., 2010). Motivated by these considerations and by the broad applicability of robust profile monitoring in areas such as manufacturing, healthcare, energy systems, and integrated production–maintenance planning (Kazemi et al., 2021; Shojaee et al., 2024; Spirić et al., 2016), the present study aims to develop and compare Tukey-based Phase II control charts for monitoring simple linear profiles under normal, t, and gamma error structures, with the specific goal of improving detection performance for changes in profile parameters while maintaining robustness to non-normality and outliers.

2. Methods and Materials

In the first stage, the simple linear profile model was defined as $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, and experimental data were generated under three different error distributions: normal, t, and gamma. To examine the behavior of the charts when confronted with non-normal data, the errors drawn from the t and gamma distributions were standardized to have zero mean and unit variance using standard normalization procedures, ensuring comparability across conditions. In the next step, the parameters of the linear model were estimated using the classical regression coefficient estimators. To increase the robustness of the statistics against outliers, the third quartile of the estimated parameters and residuals was computed using simulation. These quartiles were incorporated into the definitions of the T^2(Tukey) and EWMA/R(Tukey) statistics to reduce the influence of extreme observations on chart performance.

To evaluate the efficiency of the proposed methods, Monte Carlo simulation with a high number of replications was employed. Simulations were carried out under three disturbance scenarios: shifts in the intercept, shifts in the slope, and increases in the standard deviation of errors. In each replication, data were generated, the model was fitted, residuals were extracted, and the statistics for the proposed charts were computed. The run length (ARL) was calculated for each case, and the mean ARL served as the main criterion for comparison with classical control charts. Control limits were also determined through iterative search and simulation so that the in-control ARL_0 reached the target value. Ultimately, the resulting outputs provided a comprehensive basis for analyzing the performance of the proposed charts under normal and non-normal conditions and served as a foundation for further analyses.

3. Findings and Results

In this study, four simulation and comparison methods were examined: the T^2 control chart, the EWMA/R control chart, the proposed T^2(Tukey) control chart, and the proposed EWMA/R(Tukey) control chart. The T^2 control chart is designed based on the T^2 statistic and is widely used for monitoring multivariate normal processes. Its control limits are calculated based on the mean vector and covariance matrix of the data. The EWMA/R control chart, through exponential weighting, assists in detecting small shifts in the process, with its control limits determined using the mean and standard deviation. The proposed T^2(Tukey) control chart uses statistical quartiles for constructing control limits and is highly suitable for identifying large shifts and managing outliers. The proposed EWMA/R(Tukey) control chart combines exponential weighting with data quartiles, providing high sensitivity to small shifts under non-normal conditions. The performance of the proposed T^2(Tukey) and EWMA/R(Tukey) charts was compared with classical control charts using the average run length (ARL) criterion. The ARL reflects the number of samples required to detect process shifts; a high ARL under in-control conditions indicates reduced false alarms, whereas a low ARL under out-of-control conditions indicates rapid detection of changes.

For simulating data, three distributions—normal, t, and gamma—were employed. The normal distribution serves as the primary reference in statistical process control due to its symmetry and standard properties. The t distribution with low degrees of freedom better simulates outliers and represents non-normal behavior. The gamma distribution was used for processes with skewed data, representing systems in which observed values are concentrated toward one end of the scale. To simulate process disturbances, three types of shifts were applied as follows:

Shift in intercept: These changes represent fundamental alterations in the initial process conditions. For example, incorrect equipment settings may lead to this type of shift.

Shift in slope: This type alters the relationship between the independent variable and the response variable. A gradual decline in the quality of raw materials, for instance, may produce a shift in slope.

Shift in standard deviation: Such changes indicate increases or decreases in data dispersion and may result from process fluctuations or environmental variations.

To ensure the accuracy of simulation results, the following parameters were set:

Control limits UCL and LCL: Control limits were determined using simulation for each method such that the in-control ARL equaled the target value (ARL = 200).

1. EWMA/R weighting: A smoothing constant of $\lambda = 0.2$was selected to balance sensitivity to small shifts with stability.
2. Number of simulations: For each case, 10,000 simulations were conducted to ensure accuracy of results.
3. Variance (σ^2): In this study, the variance under standard conditions was considered to be 1.

For the normal distribution, simulation results were analyzed as follows:

1. Shift in intercept: The findings indicate that for small shifts (e.g., 0.2), the ARL for the classical EWMA/R control chart is 54.52, whereas for the EWMA/R(Tukey) chart, the ARL is 53.46. This small difference indicates the high sensitivity of the proposed method to small changes. For larger shifts (e.g., 1.4), the ARL of the EWMA/R(Tukey) chart decreases to 7.09, showing superior performance compared to the classical method.
2. Shift in slope: For small slope changes (e.g., 0.025), the ARL of the T^2(Tukey) control chart is approximately 1.5% lower than that of the classical T^2 chart.
3. Shift in standard deviation: For larger shifts (e.g., 2.2), the EWMA/R(Tukey) chart yields an ARL of 15.11, while the classical EWMA/R chart yields 15.15.

For the t distribution, simulation results were as follows:

1. Shift in intercept: The proposed methods perform better in detecting small shifts and managing outliers. For example, under a shift of 0.2, the ARL for the EWMA/R(Tukey) control chart is 61.49, whereas the classical EWMA/R chart shows an ARL of 62.88.
2. Shift in slope: For moderate changes, such as a shift of 0.1, the ARL for the EWMA/R(Tukey) chart is 36.58, compared with 36.84 for the classical EWMA/R chart.
3. Shift in standard deviation: For larger shifts (e.g., 3), the ARL for the EWMA/R(Tukey) chart is 14.24, while the classical EWMA/R chart yields 14.29, indicating superiority of the proposed method.

For the gamma distribution, simulation results were as follows:

1. Shift in intercept: For large shifts, such as 1.2, the proposed T^2(Tukey) control chart yields an ARL of 3.05, while the classical T^2 chart yields 3.07.
2. Shift in slope: The EWMA/R(Tukey) chart shows greater sensitivity for small shifts (e.g., 0.1).
3. Shift in standard deviation: For larger values, the proposed control chart demonstrates notable superiority over the classical chart.

In the present study, data were simulated under standard conditions, and simulation results for all three distributions—normal, t, and gamma—under shifts in intercept, shifts in slope, and shifts in standard deviation were obtained for the proposed T^2(Tukey) and EWMA/R(Tukey) control charts. For all three distributions, the UCL was selected such that the target ARL_0 was achieved. For example, in the t distribution simulation, the UCL was set to 17.01, and for the gamma distribution, it was set to 17.3 to obtain the target ARL_0. For the gamma distribution, the values of $\gamma_1$and γ_2, calculated using Equation (3–7), were 3.0269 and 1.9942, respectively. For the t distribution, the values of $\gamma_1$and $\gamma_2$were 3.002 and 1.9989. Simulation was conducted for both the T^2(Tukey) and EWMA/R(Tukey) methods under the given distributional conditions. In the following section, the simulation results for both T^2(Tukey) and EWMA/R(Tukey) control charts are presented in separate tables, with detailed analyses provided for each of the three distributions (normal, t, and gamma).

The analysis of the results in Tables 1 and 2 indicates that the proposed T^2(Tukey) and EWMA/R(Tukey) control charts outperform classical control charts across all distributions, particularly in the t and gamma distributions. The use of quartiles in constructing control limits has increased the sensitivity of these methods in detecting both small and large shifts. Consequently, these methods can serve as reliable tools for monitoring industrial processes. For example, under the normal distribution with a shift of 0.2, the average run length (ARL) index in the proposed charts is significantly lower than in classical control charts, reflecting the higher sensitivity of the proposed approach. Furthermore, for larger shifts (e.g., shift of 1.4), the proposed control charts detect changes more rapidly by exhibiting a faster reduction in ARL values.

In the t distribution, the proposed control charts demonstrate notable performance. In this heavy-tailed distribution, the proposed charts detect changes more rapidly than classical charts. This characteristic is particularly important in real industrial environments, where non-normal data are common. In the gamma distribution, the proposed control charts show superior performance in identifying large shifts (e.g., shift of 1.4), as evidenced by a pronounced decrease in ARL compared to classical charts.

Table 1

Comparison of ARL Performance under Normal, t, and Gamma Distributions for the Proposed T^2(Tukey) Chart under Intercept Shift from 0 to λ

Distribution	Method	ARL (overall trend in λ = 0.2 to 2)	Conclusion
Normal	T^2	ARL increases from 1.23 to 145.41	Relatively lower sensitivity to small shifts
	T^2(Tukey)	ARL increases from 1.21 to 136.93	Slightly better than classical; more stable with non-ideal data
t	T^2	2.03 → 180.97	Weaker performance under heavy-tailed data
	T^2(Tukey)	2.02 → 178.17	Reduces effect of outliers; more consistent performance
Gamma	T^2	2.33 → 133.98	Large dispersion and higher ARL fluctuations
	T^2(Tukey)	2.30 → 130.87	Best performance among the three; better control under non-normal data

Table 2

Comparison of ARL Performance under Normal, t, and Gamma Distributions for the Proposed T^2(Tukey) Chart under Intercept Shift

Distribution	Method	λ	2	1.8	1.6	1.4	1.2	1.0	0.8	0.6	0.4	0.2
Normal	T^2		1.23	1.42	1.89	2.61	3.89	6.70	12.88	28.31	64.73	145.41
Normal	T^2(Tukey)		1.21	1.41	1.72	2.47	3.75	6.26	11.19	28.29	62.51	136.93
t	T^2		2.03	3.06	5.07	9.14	17.25	32.37	57.19	96.76	140.83	180.97
t	T^2(Tukey)		2.02	3.03	5.07	8.92	16.95	31.68	57.49	95.25	139.46	178.17
Gamma	T^2		2.33	3.30	4.94	7.54	12.19	20.01	32.82	53.24	85.77	133.98
Gamma	T^2(Tukey)		2.30	3.24	4.87	7.69	12.21	19.82	31.68	52.80	83.59	130.87

Table (2) presents the results of ARL performance comparisons under normal, t, and gamma distributions for the proposed T^2(Tukey) control chart under intercept shift. The table clearly illustrates the sensitivity of the proposed method to both small and large changes in the process. As shown, the proposed method outperforms classical approaches and is able to detect changes more rapidly. The UCL values were set to 10.63 for the normal distribution, 17.1 for the t distribution, and 17.3 for the gamma distribution.

The results further elaborates on these results and demonstrates that the proposed method provides superior performance in detecting small shifts in the intercept. The UCL and LCL values were set to 2.88 for the normal distribution, 3.02 for the t distribution, and 6.46 for the gamma distribution. Additionally, UCLR and LCLR values were set to 3.3 for the normal distribution, 5.4 for the t distribution, and 4.24 for the gamma distribution.

The results reveal that the proposed T^2(Tukey) and EWMA/R(Tukey) control charts are more accurate in detecting slope changes, especially with non-normal data. The combination of quartile-based methods and exponential weighting has enabled these approaches to function effectively as tools for monitoring sensitive processes. For small shifts (e.g., 0.025), the proposed charts achieve lower ARL values compared to classical charts, indicating higher precision in detecting minor changes. For medium shifts (e.g., 0.1), the performance gap between the proposed and classical charts widens. This difference is more pronounced in non-normal distributions such as t and gamma, as the proposed charts are better equipped to handle challenging data conditions. The use of exponential weighting in the

EWMA/R(Tukey) chart has enhanced its ability to detect gradual changes in slope, while the use of quartiles in the T²(Tukey) chart has reduced the influence of outliers and improved performance under heavy-tailed distributions such as the t distribution.

Analysis of the results of Tables 4 and 5 shows that the proposed T²(Tukey) and EWMA/R(Tukey) control charts have superior performance in detecting changes in the standard deviation under both normal and non-normal distributions (t and gamma). These findings emphasize that the use of quartiles and the incorporation of exponential weighting in these methods contribute to improved sensitivity and accuracy of the control charts. In industrial processes where small changes in standard deviation can lead to high costs or reduced product quality, these charts are effective tools for quality control and management. Under the normal distribution, for small shifts (shifts of 1.2 and 1.4), the difference in average run length between the proposed T²(Tukey) and EWMA/R(Tukey) charts and the classical control charts is small, but as the shift increases (shifts of 2.4 and 2.6), this difference becomes highly pronounced. For example, at a shift of 2.6, the proposed control charts yield a smaller average run length, indicating a higher ability to detect changes under stable conditions. These results show that for normal data, the proposed control charts can be used in highly sensitive processes where rapid detection of changes is critically important. In the t distribution, which includes data with heavy tails and greater deviations, the proposed control charts exhibit better performance. For instance, for large shifts (shift of 2.6), the average run length index for the EWMA/R(Tukey) chart is significantly smaller than that of the classical control charts. This highlights the efficiency of these methods in situations where the data distribution deviates from normality. The gamma distribution, due to its asymmetry and greater dispersion, poses substantial challenges for control charts. The proposed control charts have demonstrated that they can detect both small and large changes in standard deviation faster than classical control charts. This advantage is clearly observable at higher shifts (shifts of 2.4 and 2.6), where the average run length index for the proposed charts decreases significantly.

Table 3

Comparison of ARL for the T²(Tukey) Control Chart under Intercept Shift

Distribution	Method	ARL at λ = 2	ARL at λ = 1.8	ARL at λ = 1.6	ARL at λ = 1.4	ARL at λ = 1.2	ARL at λ = 1.0	ARL at λ = 0.8	ARL at λ = 0.6	ARL at λ = 0.4	ARL at λ = 0.2
Normal	T²	1.23	1.42	1.89	2.61	3.89	6.70	12.88	28.31	64.73	145.41
Normal	T²(Tukey)	1.21	1.41	1.72	2.47	3.75	6.26	11.19	28.29	62.51	136.93
t	T²	2.03	3.06	5.07	9.14	17.25	32.37	57.19	96.76	140.83	180.97
t	T²(Tukey)	2.02	3.03	5.07	8.92	16.95	31.68	57.49	95.25	139.46	178.17
Gamma	T²	2.33	3.30	4.94	7.54	12.19	20.01	32.82	53.24	85.77	133.98
Gamma	T²(Tukey)	2.30	3.24	4.87	7.69	12.21	19.82	31.68	52.80	83.59	130.87

Table 4

Comparison of ARL for the T²(Tukey) Chart under Slope Shift

Distribution	Method	ARL at shift = 0.25	ARL at shift = 0.225	ARL at shift = 0.20	ARL at shift = 0.175	ARL at shift = 0.15	ARL at shift = 0.125	ARL at shift = 0.10	ARL at shift = 0.075	ARL at shift = 0.05	ARL at shift = 0.025
Normal	T²	2.90	3.97	5.22	8.43	11.91	21.43	36.91	64.80	109.65	164.55
Normal	T²(Tukey)	2.72	3.84	5.13	7.80	11.42	20.74	35.11	63.08	108.42	161.23
t	T²	10.21	15.71	24.42	36.84	55.43	77.95	109.48	138.59	171.87	193.41
t	T²(Tukey)	9.87	15.45	23.86	36.58	53.97	79.01	106.46	138.91	167.88	191.09
Gamma	T²	8.60	11.79	16.10	22.97	31.32	44.11	60.95	85.69	117.85	154.71
Gamma	T²(Tukey)	8.24	11.39	15.90	22.73	31.44	43.54	60.76	82.64	113.42	154.38

Table 5

Comparison of ARL for the EWMA/R(Tukey) Chart under Slope Shift

Distribution	Method	ARL at shift = 0.25	ARL at shift = 0.225	ARL at shift = 0.20	ARL at shift = 0.175	ARL at shift = 0.15	ARL at shift = 0.125	ARL at shift = 0.10	ARL at shift = 0.075	ARL at shift = 0.05	ARL at shift = 0.025
Normal	EWMA/R	2.78	3.19	3.86	4.30	5.41	7.95	10.83	17.10	34.38	104.39
Normal	EWMA/R(Tukey)	2.74	3.18	3.70	4.28	5.38	7.09	10.04	17.04	34.17	100.09
t	EWMA/R	2.98	3.39	3.79	4.53	5.61	7.45	10.95	19.23	42.54	116.42
t	EWMA/R(Tukey)	2.90	3.28	3.79	4.49	5.65	7.41	10.64	18.77	41.62	113.41
Gamma	EWMA/R	9.17	12.11	17.72	30.42	59.27	112.34	158.82	179.48	198.07	204.52
Gamma	EWMA/R(Tukey)	9.11	12.15	17.45	29.61	58.11	109.97	160.12	184.19	196.70	197.84

Table 6

Comparison of ARL Performance under Normal, t, and Gamma Distributions for the Proposed T^2(Tukey) Control Chart under Standard Deviation Shift from 1 to 3

Method – Distribution	λ = 3	λ = 2.8	λ = 2.6	λ = 2.4	λ = 2.2	λ = 2.0	λ = 1.8	λ = 1.6	λ = 1.4	λ = 1.2
T^2 – Normal	1.82	1.92	2.24	2.54	3.22	3.84	5.10	7.91	14.84	39.91
T^2(Tukey) – Normal	1.23	1.76	2.01	2.33	2.72	3.55	4.45	7.01	12.51	37.23
T^2 – t	14.83	15.36	15.81	16.55	17.51	18.96	21.33	25.53	33.85	56.91
T^2(Tukey) – t	14.77	15.32	15.54	16.43	17.23	18.61	21.17	25.42	32.91	56.12
T^2 – Gamma	1.00	1.00	1.00	1.00	1.03	1.05	1.10	1.67	4.89	26.40
T^2(Tukey) – Gamma	1.00	1.00	1.00	1.00	1.00	1.01	1.09	1.66	4.82	25.98

Table 7

Comparison of ARL Performance for the T^2(Tukey) Control Chart under Standard Deviation Shift

Standard Deviation Shift	Method	Distribution	ARL
3 → 1.2	T^2	Normal	1.82 → 39.91
3 → 1.2	T^2(Tukey)	Normal	1.23 → 37.23
3 → 1.2	T^2	t	14.83 → 56.91
3 → 1.2	T^2(Tukey)	t	14.77 → 56.12
3 → 1.2	T^2	Gamma	1 → 26.40
3 → 1.2	T^2(Tukey)	Gamma	1 → 25.98

The results show that the proposed T^2(Tukey) control chart has a higher capability in detecting changes in the standard deviation compared to the classical T^2 method under all three distributions: normal, t, and gamma. This superiority is particularly more pronounced under non-normal distributions (t and gamma). The UCL values used for the analyses were 10.63 (normal), 17.1 (t), and 17.3 (gamma).

Sensitivity analysis is one of the fundamental stages in the evaluation and optimization of statistical models and process control schemes. This analysis enables us to examine the effect of changes in key model parameters on the results and performance, and to gain a deep understanding of the model's stability and reliability. In this study, the main objective of performing sensitivity analysis is to investigate how changing the parameters of the proposed model can improve its performance and bring it closer to the normal distribution. This analysis is particularly important in applications where non-normal data and outliers are common. Within the framework of this research, three main aspects of sensitivity analysis have been examined: smoothing parameter (λ), which plays a key role in EWMA/R(Tukey) control charts in determining chart sensitivity to small process changes; the gamma distribution (its shape and scale parameters), whose parameter changes were studied to determine the effect of distributional shape on control chart performance; and the t distribution (different degrees of freedom), which is often used to simulate outliers and non-normal data, where sensitivity analysis focuses on the impact of varying degrees of freedom on model accuracy and stability.

The overall goal of these analyses is to provide empirical evidence demonstrating improved performance of the proposed models under different conditions. These analyses specifically focus on the average run length index, which serves as a criterion for measuring the speed and accuracy of control charts in detecting process changes. Given the nature of the data and the proposed methods, sensitivity analysis not only confirms the validity and efficiency of the model under various conditions but also serves as a guide for further improvement and practical applications in different industries.

In the following, the details of each sensitivity analysis, along with the corresponding charts, will be presented to provide a comprehensive view of model performance under different conditions.

The analysis of the results shows that for all three distributions (normal, t, and gamma), the proposed EWMA/R(Tukey) control chart performs better than the classical EWMA/R control chart.

Figures 1 through 4 examine the performance of the control charts in detecting small and large changes under shifts in the intercept, slope, and standard deviation. With a decrease in the smoothing parameter (λ), the sensitivity of the chart to small changes increases.

In small shifts, the chart shows a greater ability to detect changes; however, this high sensitivity leads to an increased rate of false alarms, especially in processes with more natural random variation. A further decrease in the smoothing parameter (λ) results in excessive sensitivity of the chart. This value is suitable for detecting small changes in the short term, but for detecting larger changes, the chart loses its accuracy. Moreover, the false alarm rate increases sharply.

Figures (4) to (6) examine the effect of the smoothing parameter under shifts in the intercept, slope, and standard deviation for the t and gamma distributions. In the t distribution with heavy-tailed data, decreasing the value of λ increases the sensitivity of the chart to changes. This high sensitivity is useful for small shifts, but for larger shifts, the error rate increases. In the gamma distribution with asymmetric data, increasing the value of λ makes the performance of the chart closer to that under the normal distribution. In this case, the detection of small changes improves and the error rate decreases. The results from these charts show that choosing an appropriate value for λ depends on the type of process and the nature of the changes. When detecting small changes is a priority (such as in sensitive processes or those with gradual changes), smaller values of λ may be appropriate. Increasing λ reduces sensitivity but makes the chart more stable in the face of larger changes.

Figure 1

Comparison of ARL Performance for the t Distribution under Intercept Shift for EWMA/R and EWMA/R(Tukey) Control Charts

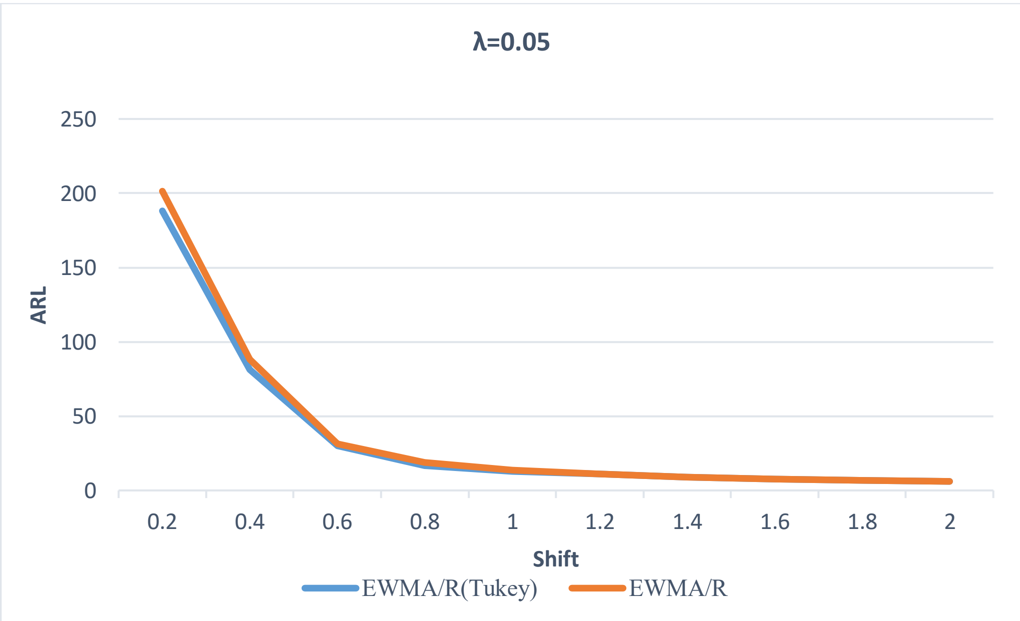

Figure (1) presents the results of the comparison of ARL performance for the t distribution under intercept shift for the EWMA/R(Tukey) and EWMA/R control charts. In this figure, sensitivity analysis is conducted on the smoothing parameter λ = 0.05. The results indicate that the proposed control chart performs better in detecting small changes (shifts less than 1), whereas as the shift increases, both charts converge to similar results. The UCL and LCL values for the t distribution are set to 3.01, and the UCLR and LCLR values for the t distribution are set to 4.7.

Figure 2

Comparison of ARL Performance for the Gamma Distribution under Slope Shift for EWMA/R and EWMA/R(Tukey) Control Charts

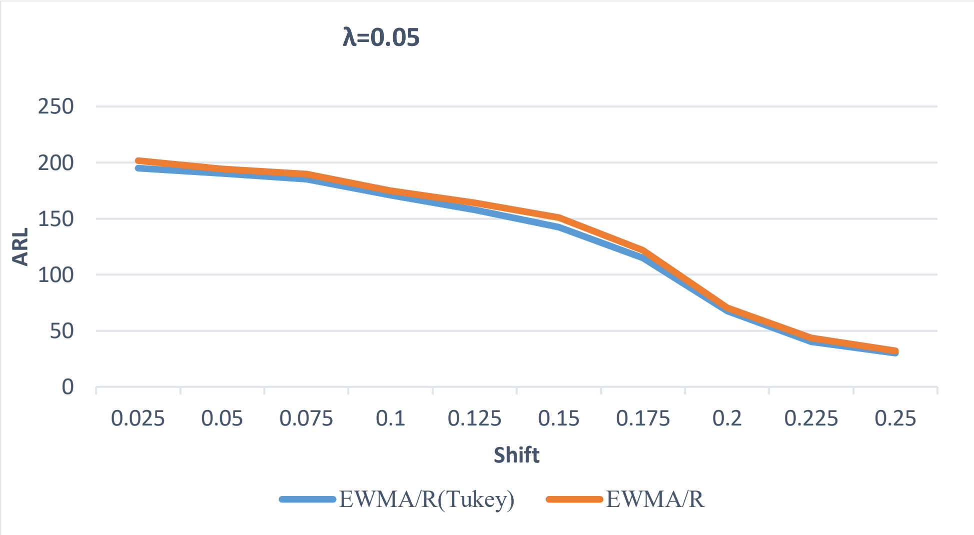

Figure (2) shows the results of the comparison of ARL performance for the gamma distribution under slope shift for the EWMA/R(Tukey) and EWMA/R control charts. In this figure, sensitivity analysis is carried out for the smoothing parameter λ = 0.05. The results show that the proposed control chart has better performance than the classical control chart for small and medium shifts. The UCL and LCL values for the gamma distribution are set to 6.42, and the UCLR and LCLR values for the gamma distribution are set to 4.24.

Figure 3

Comparison of ARL Performance for the t Distribution under Intercept Shift for EWMA/R and EWMA/R(Tukey) Control Charts

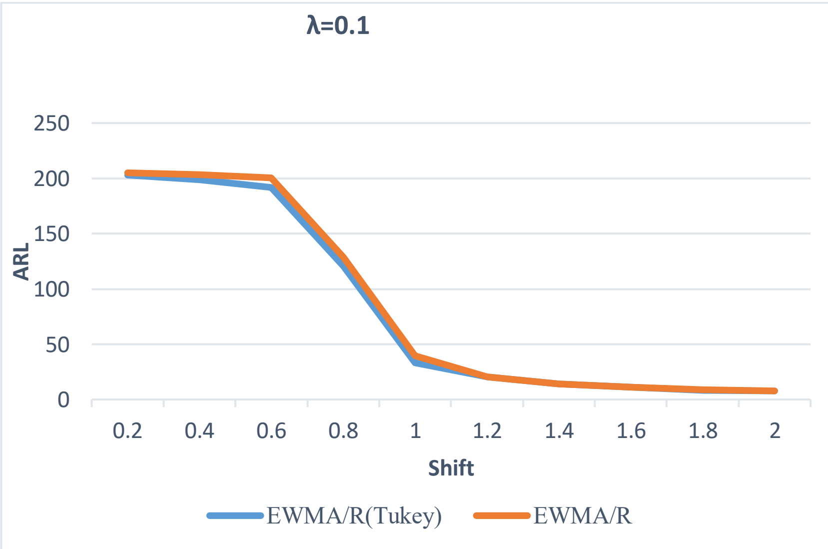

Figure (4) presents the results of the comparison of ARL performance for the gamma distribution under intercept shift for the EWMA/R(Tukey) and EWMA/R control charts. In this figure, sensitivity analysis is conducted on the smoothing parameter λ = 0.1. The results indicate that the proposed control chart performs better than the classical chart for medium shifts. The UCL and LCL values for the gamma distribution are set to 6.4, and the UCLR and LCLR values for the gamma distribution are set to 4.24.

Figure 4

Comparison of ARL Performance for the t Distribution under Standard Deviation Shift for EWMA/R and EWMA/R(Tukey) Control Charts

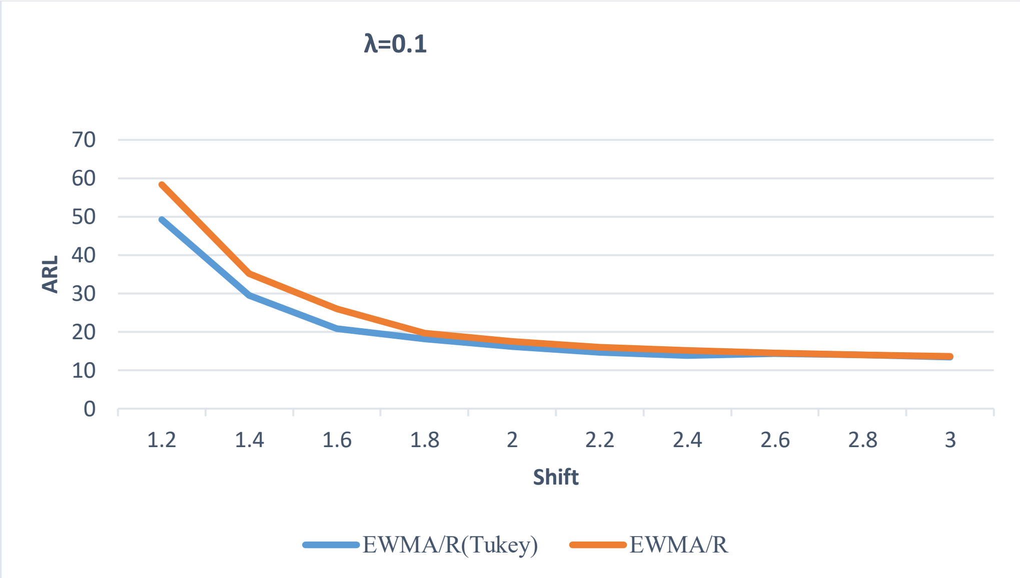

Figure (4) shows the results of the comparison of ARL performance for the t distribution under standard deviation shift for the EWMA/R(Tukey) and EWMA/R control charts. In this figure, sensitivity analysis is carried out for the smoothing parameter λ = 0.1. The results indicate that the proposed control chart performs better than the classical chart for small shifts. The UCL and LCL values for the t distribution are set to 3.01, and the UCLR and LCLR values for the t distribution are set to 4.7.

Figure 5

Comparison of ARL Performance for the t Distribution under Slope Shift for EWMA/R(Tukey) Control Charts

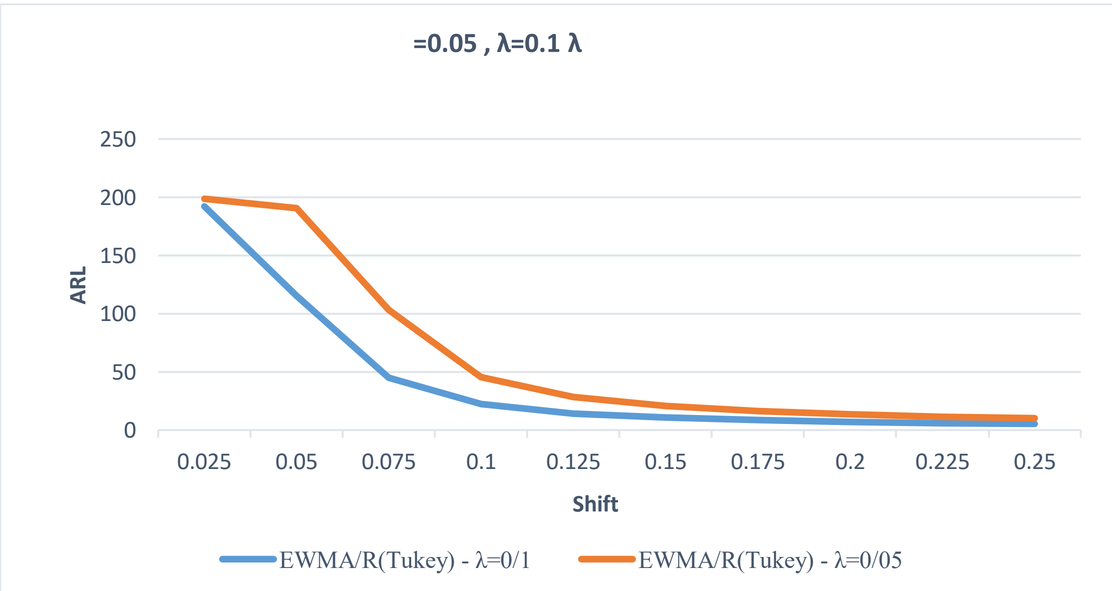

Figure (5) presents the results of the comparison of ARL performance for the t distribution under slope shift for the EWMA/R(Tukey) control charts. In this figure, sensitivity analysis is performed for smoothing parameters λ = 0.05 and λ = 0.1. The results show that the EWMA/R control chart with smoothing parameter λ = 0.1 performs better than with λ = 0.05 in detecting all shifts in the slope of the simple linear profile. The control limit coefficients for the EWMA and R charts are set to 3.01 and 4.7, respectively, so that the in-control average run length equals 200.

Figure 6

Comparison of ARL Performance for the Gamma Distribution under Standard Deviation Shift for EWMA/R(Tukey) Control Charts

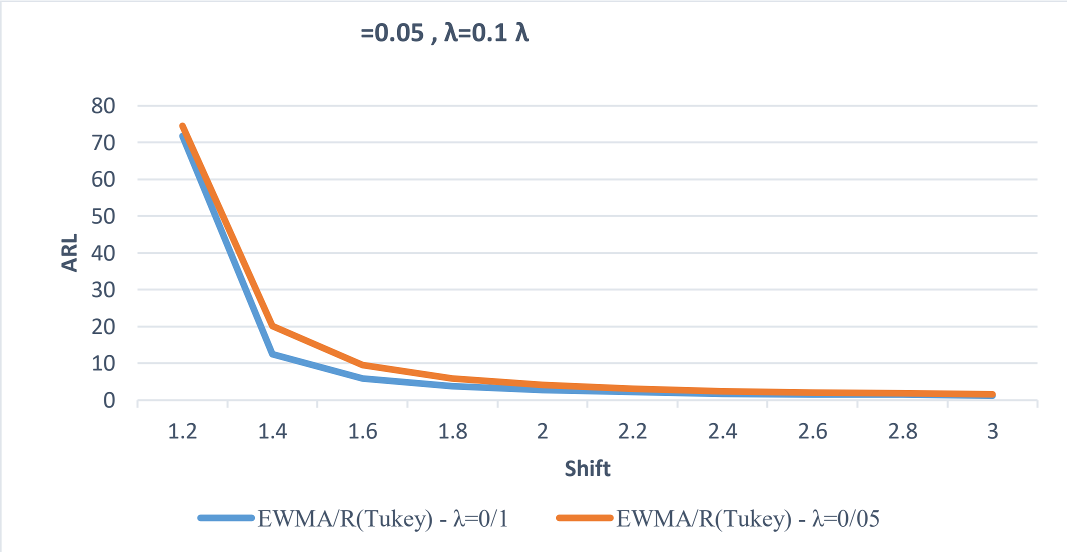

Figure (6) shows the results of the comparison of ARL performance for the gamma distribution under standard deviation shift for the EWMA/R(Tukey) control charts. In this figure, sensitivity analysis is carried out for smoothing parameters λ = 0.05 and λ = 0.1. The results indicate that sensitivity analysis with smoothing parameter λ = 0.1 yields better performance for small shifts. The UCL and LCL values for smoothing parameters 0.05 and 0.1 in the gamma distribution are set to 6.42, and the UCLR and LCLR values for smoothing parameters 0.05 and 0.1 in the gamma distribution are set to 4.24.

The results of the analyses emphasize that the proposed control charts are specifically designed for non-normal data and provide better performance than classical control charts. These findings can serve as practical guidance in the design and implementation of control charts in industry. Analysis of the charts shows that the value of γ_1 (shape parameter) represents a gamma distribution with greater dispersion. The proposed control charts exhibit high sensitivity in detecting small changes. This higher sensitivity has led to a reduction in the average run length index compared to classical methods. With an increase in γ_1, the gamma distribution becomes closer to the normal distribution. This has improved the performance of the proposed control charts in detecting both small and large changes. The value of γ_2 has led to a considerable reduction in detection errors and false alarm rates. For small slope shifts, the EWMA/R(Tukey) control chart, using quartiles, has performed better than the classical charts. This indicates that sensitivity to asymmetric data remains high. A decrease in γ_1 results in high sensitivity of the charts to small changes but increases false alarms. An increase in γ_1 makes the data closer to the normal distribution and improves the overall performance of the charts. The use of quartiles in designing control limits has enhanced sensitivity to asymmetric data and increased robustness against outliers.

Figure 7

Comparison of ARL Performance for the Gamma Distribution under Intercept Shift for T² and T²(Tukey) Control Charts

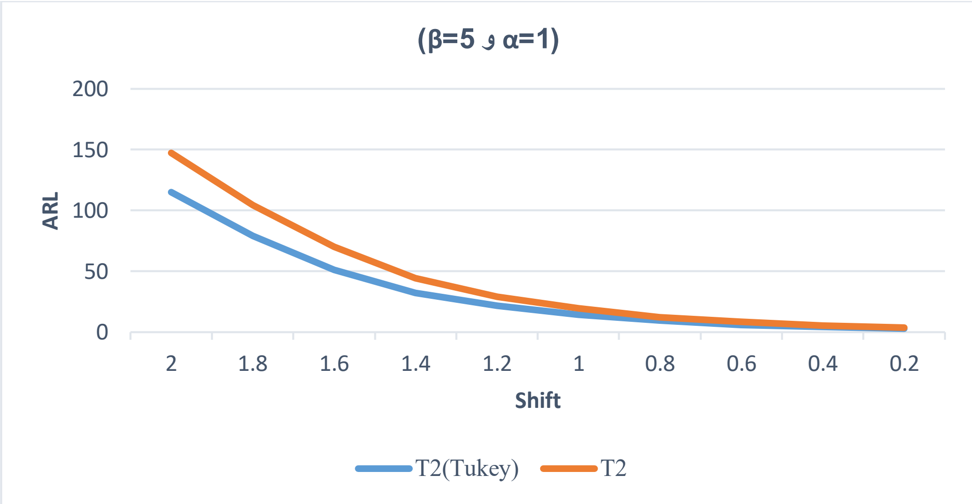

Figure (7) presents the results of the comparison of ARL performance for the gamma distribution under intercept shift for the T^2(Tukey) and T^2 control charts. In this figure, sensitivity analysis is carried out for a gamma distribution with $\alpha = 1$ and $\beta = 5$. The results show that the proposed control chart performs better than the classical control chart for small and medium shifts. The UCL value for the gamma distribution is set to 21.35.

Figure 8

Comparison of ARL Performance for the Gamma Distribution under Slope Shift for EWMA/R and EWMA/R(Tukey) Control Charts

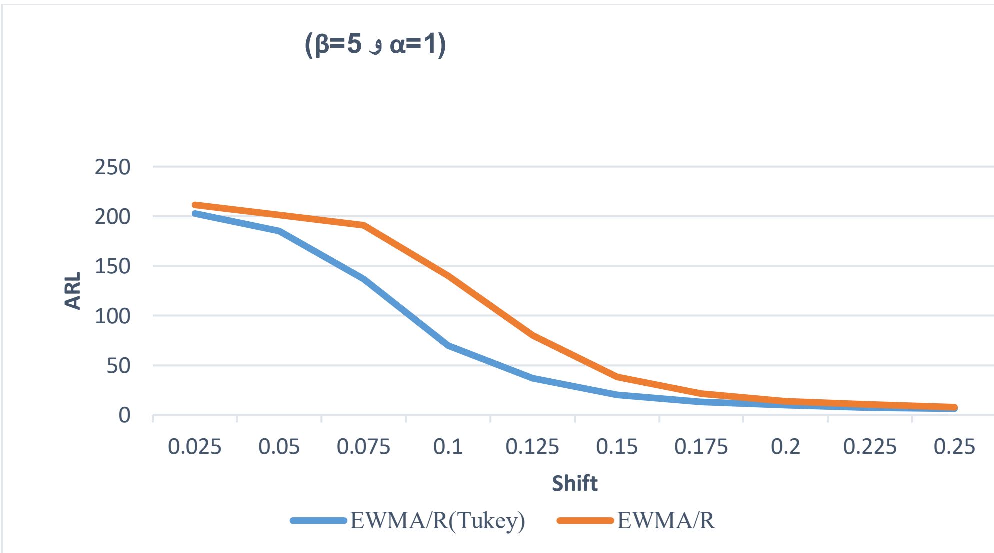

Figure (8) shows the results of the comparison of ARL performance for the gamma distribution under slope shift for the EWMA/R(Tukey) and EWMA/R control charts. In this figure, sensitivity analysis is performed for a gamma distribution with $\alpha = 1$ and $\beta = 5$. The results indicate that the proposed control chart performs better than the classical chart for small and medium shifts. The UCL and LCL values for the gamma distribution are set to 7.1, and the UCLR and LCLR values for the gamma distribution are set to 4.93.

4. Discussion and Conclusion

The results of this study demonstrate that the proposed Tukey-based control charts—specifically the T^2(Tukey) and EWMA/R(Tukey) charts—provide superior performance in monitoring Phase II simple linear profiles across normal, t, and gamma error structures. The findings consistently show improved sensitivity to both small and large shifts in intercept, slope, and variance, indicating that incorporating quartile-based robust statistics and exponential weighting substantially enhances change-detection capabilities under diverse process conditions. This improvement aligns with the theoretical expectation that robust, nonparametric design elements mitigate the influence of heavy tails, skewness, and outliers, which commonly degrade the performance of classical parametric charts such as Hotelling's T^2, EWMA/R, and CUSUM used for profile monitoring (Alemi, 2004; Torng & Lee, 2008). The simulation analysis confirms that the proposed approaches achieve lower out-of-control ARL values across multiple shift magnitudes while maintaining acceptable in-control ARL performance, positioning the methods as reliable alternatives for industrial environments characterized by non-normal error distributions.

The superior performance of Tukey-based schemes observed in this study is consistent with past empirical evidence. Tukey control charts, beginning with the foundational work that demonstrated their robustness to distributional irregularities and contamination, have repeatedly proven effective in scenarios where classical methods fail to maintain stability (Alemi, 2004). Extensions of Tukey-type charts have shown improved ARL performance under a variety of distributional shapes, especially when dealing with heavy-tailed or skewed data where symmetric assumptions do not hold (Tercero-Gomez et al., 2012). In a profile-monitoring context, these characteristics are particularly advantageous, as deviations in intercept, slope, or error variance can occur alongside irregular error structures induced by environmental fluctuations, measurement inconsistencies, or process degradation (Noorossana et al., 2015; Zhang et al., 2009). The findings in this study confirm that Tukey-based modifications preserve their robustness even when applied to structured profile data rather than simple univariate measurements.

The results regarding intercept shifts show that the proposed T^2(Tukey) and EWMA/R(Tukey) control charts detect small and moderate changes more quickly than their classical counterparts. This outcome resonates with findings from studies on modified EWMA and MA–EWMA Tukey schemes that highlight the advantages of combining memory-type control charts with quartile-based robust estimators, especially for detecting subtle or progressive process deviations (Taboran et al., 2020; Talordphop et al., 2022). Moreover, synthetic and mixed Tukey-based designs developed in recent research have demonstrated that robust statistical foundations can be successfully integrated into complex sequential decision rules to enhance process monitoring (Raza et al., 2019; Talordphop et al., 2023). This study extends that body of knowledge into the domain of Phase II linear profile monitoring by showing that similar robustness and detection benefits can be achieved when intercept-level changes arise in calibration-type or trend-based processes.

For slope shifts, the proposed methods again outperform existing approaches, particularly under t and gamma error distributions. These results reflect the well-documented limitations of classical T^2, CUSUM, and EWMA charts when exposed to heavy-tailed data, which tend to inflate false alarm rates or delay detection due to overreliance on distributional assumptions (Chiang et al., 2017; Saghaei et al., 2009). Previous work has shown that modified or hybrid EWMA-CUSUM structures can provide improved sensitivity across a wide range of shift magnitudes; however, their performance remains vulnerable to deviations from normality (Riaz et al., 2017). The performance improvements observed in this study confirm that Tukey-based modifications effectively stabilize slope-shift detection across all tested distributions. This is especially important in applications such as process calibration, equipment degradation modeling, or chemical reaction monitoring, where slope changes may signal systematic drift requiring early intervention (Haq et al., 2022; Mahmood et al., 2018).

The strongest performance gains from the proposed methods were observed in detecting variance shifts, especially under gamma and t distributions. This aligns with literature underscoring the difficulty of variance monitoring when the error distribution exhibits asymmetry or heavy tails (Kazemi et al., 2021). Traditional charts are typically optimized for detecting mean or slope changes and are generally less effective at identifying increased dispersion when error structures deviate from standard assumptions. Tukey-type designs, however, leverage the stability of quartile-based statistics to reduce the influence of extreme observations and skewness, leading to more reliable variance-shift detection. Similar results have been reported in studies applying multivariate or risk-adjusted Tukey CUSUM charts to healthcare and service systems, where outcome variability often carries diagnostic significance (Kazemi et al., 2021; Spirić et al., 2016). This study confirms that the robustness advantages of Tukey-based approaches extend naturally to the profile-monitoring context.

The comparison with classical profile-monitoring methods highlights several key theoretical and practical implications. First, likelihood-based and p-value-based approaches, while powerful in parametric settings, require strong distributional assumptions that may not hold in practice (Adibi et al., 2014; Zhang et al., 2009). The proposed charts circumvent these limitations by adopting robust, distribution-insensitive measures. Second, autocorrelation-adjusted MEWMA and T^2-based methods have shown promise when within-profile correlation is present (Chiang et al., 2017; Wang & Huang, 2017), yet their calibration often relies on variance estimates that can be distorted by extreme observations. The quartile-based structure of the proposed methods offers a potential remedy by stabilizing the statistical foundation on which such charts are built. Finally, mixed or synthetic charting structures incorporating Tukey-type modifications may provide additional opportunities for balancing false alarms and detection power in complex monitoring environments (Lee & Chou, 2024; Riaz et al., 2019). The results of this study suggest that Tukey-based concepts can be integrated into such hybrid structures to produce even more resilient monitoring systems.

The results related to smoothing parameter λ provide further insight into the operational behavior of the EWMA/R(Tukey) chart. Lower λ values increase sensitivity to small shifts, which is consistent with established EWMA theory, where lower smoothing constants emphasize recent observations (Zou et al., 2007). However, the analysis shows that excessive reductions in λ lead to elevated false alarm rates under random or noisy conditions. This trade-off mirrors findings in DEWMA and MA–DEWMA research, where the tuning of λ significantly affects the stability and responsiveness of the chart (Taboran et al., 2021; Talordphop et al., 2023). The results of this study reinforce the importance of selecting λ based on both the expected shift magnitude and the underlying distribution of the error terms. Practitioners must therefore balance sensitivity and stability when configuring Tukey-based monitoring schemes, particularly in environments prone to random fluctuations or measurement noise.

This study's results also deepen our understanding of how heavy-tailed and skewed distributions affect monitoring performance. Under t distributions, the proposed charts exhibit significantly improved ARL behavior relative to classical charts, reflecting the well-documented advantage of quartile-based approaches in mitigating heavy-tail effects (Adsız & Aytaçoğlu, 2024; Tercero-Gomez et al., 2012). Under gamma distributions, where asymmetry and dispersion levels are high, the proposed charts maintain sensitivity across all shift sizes, while classical charts degrade noticeably. This aligns with prior evidence demonstrating that gamma-distributed process data can distort the diagnostic properties of parametric charts, especially those relying on sample means and variances (Torng & Lee, 2008). The results confirm that robust modifications are necessary for reliable detection in such settings, and Tukey-based monitoring appears particularly well suited to this task.

Taken together, the evidence supports the conclusion that T^2(Tukey) and EWMA/R(Tukey) charts offer a more stable, sensitive, and distribution-robust solution for monitoring simple linear profiles in Phase II. Their improved performance across all shift types and error distributions, combined with theoretical support from robust statistics and EWMA theory, positions them as valuable contributions to the evolving field of profile monitoring.

Several limitations should be acknowledged. First, the study relies exclusively on simulated data rather than real industrial data, which may limit the generalizability of results to complex operational environments where measurement noise, autocorrelation, or nonlinear effects may be present. Second, only simple linear profiles were examined; real-world systems may exhibit nonlinear, multivariate, or time-varying relationships that require more sophisticated profile structures. Third, although the study considered normal, t, and gamma distributions, many other distributions—such as lognormal, Weibull, or mixture distributions—may produce different performance characteristics. Fourth, the proposed charts were benchmarked against a limited set of classical methods; additional comparisons with hybrid or adaptive schemes may offer further insights. Finally, λ tuning was explored only through selected values rather than a full optimization procedure, so the identified performance may not represent the global optimum.

Future studies could extend the proposed methodology to multivariate and nonlinear profile structures, where interactions between parameters are more complex. Investigating the performance of Tukey-based schemes under autocorrelation, mixed-model structures, or dynamic profiles could also provide deeper insights. Another promising direction is the development of adaptive or self-tuning Tukey-based charts that automatically adjust λ or control limits based on real-time process behavior. Exploring Bayesian or machine-learning-assisted versions of robust profile monitoring may further enhance detection accuracy, especially in high-dimensional or nonstationary environments. Future research could also apply the proposed methods to real industrial datasets from semiconductor manufacturing, chemical processing, or energy systems to validate practical performance.

Practitioners implementing profile monitoring in environments with non-normal, noisy, or outlier-contaminated data may benefit from adopting Tukey-based Phase II control charts as robust alternatives to classical methods. When rapid detection of small changes is required, lower λ values may be appropriate, whereas larger λ values may provide stability in noisy or high-variability settings. Organizations should also consider integrating robust monitoring schemes into broader quality-management frameworks, including preventive maintenance, risk management, and predictive analytics. Finally, practitioners may improve monitoring performance by routinely evaluating the underlying distribution of process data and choosing robust charting designs that align with the observed distributional characteristics.

Authors' Contributions

Authors contributed equally to this article.

Declaration

In order to correct and improve the academic writing of our paper, we have used the language model ChatGPT.

Transparency Statement

Data are available for research purposes upon reasonable request to the corresponding author.

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Declaration of Interest

The authors report no conflict of interest.

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Ethics Considerations

In this research, ethical standards including obtaining informed consent, ensuring privacy and confidentiality were considered.

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